We present meshfree methods based on Radial Basis Function (RBF) interpolation for solving partial differential equations (PDEs) on irregular domains and surfaces; such domains are of great importance in mathematical models of biological processes. First, we present a generalized high-order RBF-Finite Difference (RBF-FD) method that exploits certain approximation properties of RBF interpolants to achieve significantly improved computational complexity, both in serial and in parallel. Like all RBF-FD methods, our method requires stabilization when applied to solving PDEs. Consequently, we present a robust and automatic hyperviscosity-based stabilization technique to rectify the spectra of RBF-FD differentiation matrices. The amount of hyperviscosity is determined quasi-analytically in two stages: first, we develop a novel mathematical model of spurious solution growth, and second, we use simple 1D Von Neumann analysis to analytically cancel out these spurious growth terms. The resulting expressions for hyperviscosity are a generalization of formulas from both RBF-FD and classical spectral methods. The resulting stabilized RBF-FD method serves as a high-order meshfree framework for solving PDEs on irregular domains. Finally, we present a powerful new RBF-FD technique that allows for the solution of PDEs on surfaces using scattered nodes and Cartesian coordinate systems. In all cases, our methods achieve O(N) complexity for N nodes.